The spectra and composition of cosmic ray primaries are being studied by fitting extensive and detailed underground muon data using scaling model calculations of hadronic interactions in the atmosphere. Three types of data have been analyzed: (1) The intensity-depth curve for the total flux of muons. (2) The muon charge ratio. (3) The rates of doubles, triples, and higher multiplicities in the standard 80 m$^2$ fiducial area of the Utah detector. Several constraints on the primary spectra and composition are obtained by considering these data and implications for some proposed spectra and compositions are discussed.

1. Introduction. Cosmic ray muons arise from atmospheric decays of mesons in hadronic cascades initiated by primary cosmic rays. A reliable model of hadron interactions at high energies would allow one to deduce information about the primaries from measurements made on the muons. We have used scaling of hadronic interactions (Feynman 1969) and accelerator hadron production measurements with nuclear targets as a guide to the nuclear physics. The application of scaling to hadron-nucleus collisions and the functions which were fitted to the accelerator data are described by Mason and Elbert (1973).

The different types of data from underground muons have different implications for the primary spectrum and composition. The intensity depth curve for the total flux of muons underground has been related to the differential primary nucleon spectrum and can be used to test primary spectral shapes. High energy Utah muon charge ratio measurements (Ashley et al. 1973) and other measurements at lower energies, on the other hand, can be related to the fraction of protons in the primary flux, thus allowing the composition to be studied for transitions to pure protons or to pure alphas and heavier nuclei in the primary energy range 0.4-40 TeV. Finally the rates of doubles, triples and higher multiplicities in the standard 80 m$^2$ fiducial area of the Utah detector (Lowe et al. 1973; Mason and Elbert 1973) are sensitive to heavier-than-proton components of the primary spectrum. Monte Carlo calculations allow at least a partial separation of the effects of the primary spectrum from the composition.

2. Information from Muon Intensities. A characteristic result of scaling is that a primary nucleon spectrum with a constant spectral index yields a production spectrum of mesons with the same spectral index. The production of muons by meson decay adds an additional energy-dependent factor to the muon spectrum. This factor asymptotically falls off at high energies as E$^{-1}$. In order to solve for muons produced at any energy with the primary spectrum given, the competition between meson decay and
interaction (determined by inelastic cross sections) must be assumed as well as a production model.

In setting up the atmospheric diffusion equations which were solved for muon intensities and charge ratios, we have assumed that neutrons and protons in primary cosmic ray nuclei are equivalent to free nucleons of equal energy per nucleon. This has been observed to be approximately true in the Monte Carlo calculations of Mason and Elbert (1973). For a cosmic ray primary nucleon spectrum $dN/dE \propto E^{-\gamma}$, scaling gives the ratios (Frazer et al. 1972)

$$Z_{ab} = \pi \int_0^1 dx_0 dP_T^2 F_{ab}(x, P_T) x^{\gamma-2}$$

(1)

of the flux of produced particles b to the flux of particles a incident on air nuclei. The assumption of scaling implies that the structure functions $F_{ab}$ depend only on the transverse momentum and $x = P_L/P_{L_{\text{Max}}}$ of particle b, in the center-of-mass-system. The functions $F_{ab}$ used in the calculation are described by Mason and Elbert (1973). For $x > 0$ and high projectile energy, $x$ is approximately the ratio of secondary to projectile laboratory energies.

Ramana Murthy and Subramanian (1972a) have used a comparison of the differential cosmic ray primary and sea level muon intensities at the same energies to test scaling up to several TeV and have used scaling to deduce a primary nucleon spectrum up to 5 TeV/nucleon from the muon intensity measurements. The resulting primary spectrum was found to be close to that of Pal (1967).

We have repeated this calculation with our scaling input and including the effect of meson induced cascades. With $\gamma = 2.7$ we obtain a predicted ratio of the vertical sea level differential muon intensity to differential incident primary nucleon intensity at 1 TeV of $1.2 \times 10^{-2}$ compared to their $0.65 \times 10^{-2}$ obtained with $\gamma = 2.67$. Our higher efficiency for muon production results from higher $Z$ values, which are given in Morrison and Elbert (1973), inclusion of meson showers, a better estimate of energy transfer in meson decay, a smaller nucleon interaction length in air (77.6 gm cm$^{-1}$ compared to 90 gm cm$^{-1}$), and a larger pion interaction length in air (132 gm cm$^{-1}$ compared to 120 gm cm$^{-1}$). The muon to primary ratio is approximately proportional to the ratio $e_\pi/e_N$. Our muon to primary ratio at 1 TeV is in good agreement with the value $1.1 \times 10^{-2}$ obtained from dividing a sea level muon differential intensity at 1 TeV, obtained from fits to a survey of vertical underground muon measurements (Bergeson et al. 1973), by a primary differential intensity at 1 TeV, obtained by augmenting the proton spectrum of Ryan et al. (1972) by a contribution from $\alpha$'s and heavies obtained from lower energy composition measurements. It appears better knowledge of the cross sections and structure functions is required to obtain accurate primary flux values from muon measurements.

A better fit to a survey of vertical underground muon intensities and to vertical underground intensities inferred from Utah inclined measurements has been obtained with a muon spectrum which steepens compared to one derived with scaling from a constant spectral index (Bergeson et al. 1973). This may be interpreted as arising from a steepening of the primary spectrum at high energies.
One of the results of the Monte Carlo calculation of Mason and Elbert (1973) using $\gamma = 2.7$ was an apparently constant value of 4 for the probability distribution of $E_i/E_p$ over the range $0 < E_i/E_p < 0.08$ for muons of energy $E_i$ produced by primary protons of energy $E_p$. A primary intensity cut-off at an energy per nucleon $E_o$ would, therefore, result in an added factor $(1 - 4 E_i/E_o)$ in the differential muon spectrum. A fit to the Utah muon intensity data with a factor of this form indicated a primary cut-off at $E_o = 200 \pm 70$ TeV. The observed muon depth intensity curve could also result from a violation of scaling, an increase of the pion cross section (Cheng and Wu 1970) or an increase of muon energy loss above expectations at high energies.

3. Information from the Charge Ratio. The high energy Utah underground muon charge ratio measurements made by Ashley et al. (1973) and a compilation of charge ratio results obtained with surface experiments have been compared to a theoretical calculation by Morrison and Elbert (1973). The calculation assumes Feynman scaling, that nuclear target effects are not negligible, and that the differential spectral slopes of all significant primary particles are $\gamma = 2.7$. The primary composition is assumed to be that measured at lower energies. The predictions are found to be in agreement with the measurements within the expected errors of both over the entire range of median primary energies (0.2-40 TeV/nucleon) for the measurements considered.

It is interesting to find what limits are implied by this result for the primary spectrum and composition in this energy range. As examples we consider whether the result is consistent with the proton primary spectrum measurements of Grigorov et al. (1970) and with an extrapolation to higher energies of the composition model recently suggested by Ramaty et al. (1973).

The predicted charge ratio is moderately sensitive to the value assumed for the spectral index, an increase of $\gamma$ by 0.1 increases the charge ratio by 0.02. The calculated charge excess $(N_i^+ - N_i)/((N_i^+ + N_i^-)$ of muons is nearly proportional to the proton excess $(N_p - N_n)/((N_p + N_n)$ when incident nuclei are treated as made of free nucleons. Finally, the charge ratio of muons from pions created by proton interactions is equal to $Z_{p\pi^+}/Z_{p\pi^-}$.

A simplified form of the charge ratio calculation can be used to estimate the sensitivity of the calculated charge ratio to a variation of the assumptions. It is assumed that muons are produced only by pions from the first interaction of primaries in the atmosphere. This is only approximately true (Morrison and Elbert 1973). In this model the charge ratio is given by

$$R = \frac{N_i^+}{N_i^-} \approx \frac{N_p Z_{p\pi^+} + N_n Z_{n\pi^-}}{N_p Z_{p\pi^-} + N_n Z_{n\pi^+}} = \frac{1 + \delta \epsilon}{\delta + \epsilon}$$  \hspace{1cm} (2)$$

The calculated ratio $\delta^{-1} = Z_{p\pi^+}/Z_{p\pi^-}$ is 1.46 for $\gamma = 2.7$, and would be expected to be energy dependent if a major violation of scaling were present. The ratio of incident neutrons to incident protons is $N_n/N_p = \bar{\alpha}$ and is taken to be about 0.1 at lower energies as in Morrison and Elbert (1973). The approximate model gives $R = 1.35$. 
The experimental charge ratio results considered by Morrison and Elbert (1973) agree with the measurements to better than 0.1 over the range corresponding to primary energies from 0.4 TeV to 20 TeV/nucleon. For small changes of the input parameters with energy occurring within the energy range of interest, the effect on the predicted charge ratios is given by

\[ \Delta R = \frac{\partial R}{\partial \delta} \Delta \delta + \frac{\partial R}{\partial \epsilon} \Delta \epsilon + \frac{\partial R}{\partial \gamma} \Delta \gamma + \frac{\partial R}{\partial \epsilon_{sv}} \Delta \epsilon_{sv} \]  \hspace{1cm} (3)

The dependence of \( \tilde{\alpha} \) on \( \gamma \) is shown in Eq. 1. Assuming primary protons make up most of the primary nucleon flux, the \( \tilde{\alpha} \) refers to possible changes in the spectral slope of protons. The last term allows for an energy dependence of \( \tilde{\alpha} \) in violation of the scaling assumption. The values of \( R / \tilde{\alpha}, R \tilde{\alpha}, \) and \( \gamma \) given by the simple model are -0.84, -1.59, and -0.126 respectively. Thus for primary energies per nucleon \( E_p \) such that 0.2 < \( E_p \) < 20 TeV,

\[ |\Delta R| = |0.20 \Delta \gamma - 0.84 \Delta \delta - 1.59 \Delta \epsilon_{sv}| \lesssim 0.1 \]  \hspace{1cm} (4)

This is the general form of the result for small changes of the independent variables but certain particular cases are of interest. If \( \tilde{\alpha} \) = 0 (scaling assumption) and the spectral slope of protons is constant in this energy range, then 0 < \( \tilde{\alpha} \) < 0.25 (this limit was obtained using Eq. 2 rather than Eq. 4). This just barely includes the possibility of a transition from our assumed spectrum to a spectrum of pure protons within the energy range considered. A pure proton primary flux would raise the charge ratio by approximately 0.1.

Recently, Ramaty et al. (1973) have proposed that the primaries of the iron group result from a different source mechanism than p's, \( \tilde{\alpha} \)'s, and CNO group particles. According to them between about 5 and 50 GeV/nucleon the spectral index of the iron group is lower than the other particles mentioned above by about 0.52. Extrapolating this behavior to 0.4 and 20 TeV/nucleon, the charge ratio would decrease by about 0.13 in the energy range of interest due to the increasing value of \( N_\alpha / N_p \) caused by the increasing importance of the Fe group at higher energies. This argues against extrapolation of the low energy result up to energies of >20 TeV/nucleon (\( E > 10^{15} \) eV/particle for iron nuclei).

The measurements of Grigorov et al. (1970) indicate that the proton spectral index increases by about 0.7 for primary energies exceeding a few TeV. This produces an increase in \( N_\alpha / N_p \) by a factor of about 4 between \( E_1 = 2 \) TeV/nucleon and \( E_2 = 20 \) TeV/nucleon. Besides the resulting change in \( \tilde{\alpha} \) arising in this energy range, \( \tilde{\alpha} \) is reduced throughout this range because of the change of \( \gamma \). Assuming that scaling is valid and \( N_\alpha / N_p = 5\% \) at \( E_1 \), the calculated charge ratio at \( E_1 \) is \( R_1 = 1.53 \) and it becomes \( R_2 = 1.32 \) at \( E_2 \). The corresponding results are \( R_1 = 1.43 \) and \( R_2 = 1.21 \) if \( N_\alpha / N_p \) is assumed to be 10\% at \( E_1 \). This behavior disagrees with the measurements. If the agreement between calculation and experiment observed at energies well below the proposed change in the proton spectral index is ignored, \( \tilde{\alpha} \) can be obtained from the value \( R_1 = 1.27 \) obtained from the data at \( E_1 \). Using 5-10\% for \( N_\alpha / N_p \) at \( E_1 \) gives \( R_2 = 1.15-1.17 \) at \( E_2 \). These values for \( R_2 \) are low when compared to the data at \( E_2 \).

The above conclusions also result from some interaction models other than scaling. For example, Morrison and Elbert (1973) describe models in which \( <n_\pi> \propto E^{\tilde{\alpha}} \), \( \tilde{\alpha} > 0 \), for which the ratio \( \tilde{\alpha}^{-1} = Z_{p\pi}^+/Z_{p\pi} \) would decrease.
with increasing energy. The disagreement between the muon charge ratio and that predicted using the spectrum of Grigorov et al. (1970) would then be more severe, and extrapolated results of Ramaty et al. (1973) would be in greater disagreement with the data.

4. Information from Multiple Muon Data. Additional evidence that the primary spectrum does not change radically between 1-1000 TeV comes from the "multiple" muon data of the Utah underground experiment. This consists of events in which one or more muons from the same cosmic ray primary are detected simultaneously. Using a scaling model, Mason and Elbert (1973) have calculated rates for detected multiplicities, nD, from one to five. The data are analyzed according to zenith angle and depth underground and various multiplicities can be associated by the calculation with median proton primary energies in the range from 10 to 1000 TeV. Assuming scaling to be correct, these data are sensitive to the slope and composition of the primary spectrum. Evidence supporting the application of the scaling assumption to the Utah underground muon data is given by Mason and Elbert (1973) and Morrison and Elbert (1973).

Results of the scaling calculation show that effects of the spectral slope are separable from the effects of the composition. For roughly equal spectral slopes of primary á's and p's, the ratio of nD = 3 events to nD = 1 events is approximately 3 or 4 times greater for primary á's than for primary protons and this factor is approximately independent of depth and angle. This is illustrated in Figure 1 of the paper by Mason and Elbert which is submitted to this conference. Thus, an analysis of the depth dependence of the data at fixed nD is relatively insensitive to the á / p value, while it is sensitive to the spectral slopes of the primaries. On the other hand, a comparison of nD = 1 event rates to nD = 3 rates at depths corresponding to approximately the same median proton energy is relatively insensitive to the proton spectral slope, but...
is sensitive to the $\alpha/p$ value. Both types of comparison are involved in the analyses in which we fit the data at various $n_D$, depth, and zenith angle values with spectral slopes and $\alpha/p$ values. Consequently, these analyses are separately sensitive to the spectral slopes and composition of the primary particles. Restricting ourselves to consideration of the spectrum on the range 1-1000 TeV, we find good agreement (reduced $\chi^2$ less than 1.0) between observed ratios of multiple muon event rates to single muon event rates and predictions based on the assumptions that the primary protons and alphas have a differential spectral index, $\gamma = 2.7$, and an alpha-to-proton ratio at the same energy per nucleon of .042 (Ryan et al. 1972). The effect of heavier primaries, which would be greatest for larger multiplicities than considered here, has not yet been assessed. We have also compared our data to predictions based on a primary spectrum of the type proposed by Grigorov et al. (1970), for which the differential spectral index is 2.62 below a transition region near 1.5 TeV and changes to 3.32 above it. The alpha (and heavier) component of the spectrum then assumes a more important role in predicting rates of muons on the energy range of interest to the Utah detector. The Grigorov spectrum, with a derived alpha-to-proton ratio of 0.0595 at 1.0 TeV / nucleon, yields a reduced $\chi^2$ of 2.9 when predictions of that spectrum are compared to our data. In fact, a fit of the spectral index (taken to be the same for protons and alphas) and the alpha-to-proton ratio at 1.0 TeV yields $\gamma = 2.62 \pm .13$ and an alpha-to-proton ratio of 0.01 $\pm .05$. Such a fit yields a reduced $\chi^2$ of 0.97. In Fig. 1 we show ratios of our data to the predictions of a scaling model for a spectrum with constant spectral index, $\gamma = 2.62$, and above it is a similar plot for the Grigorov spectrum.

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